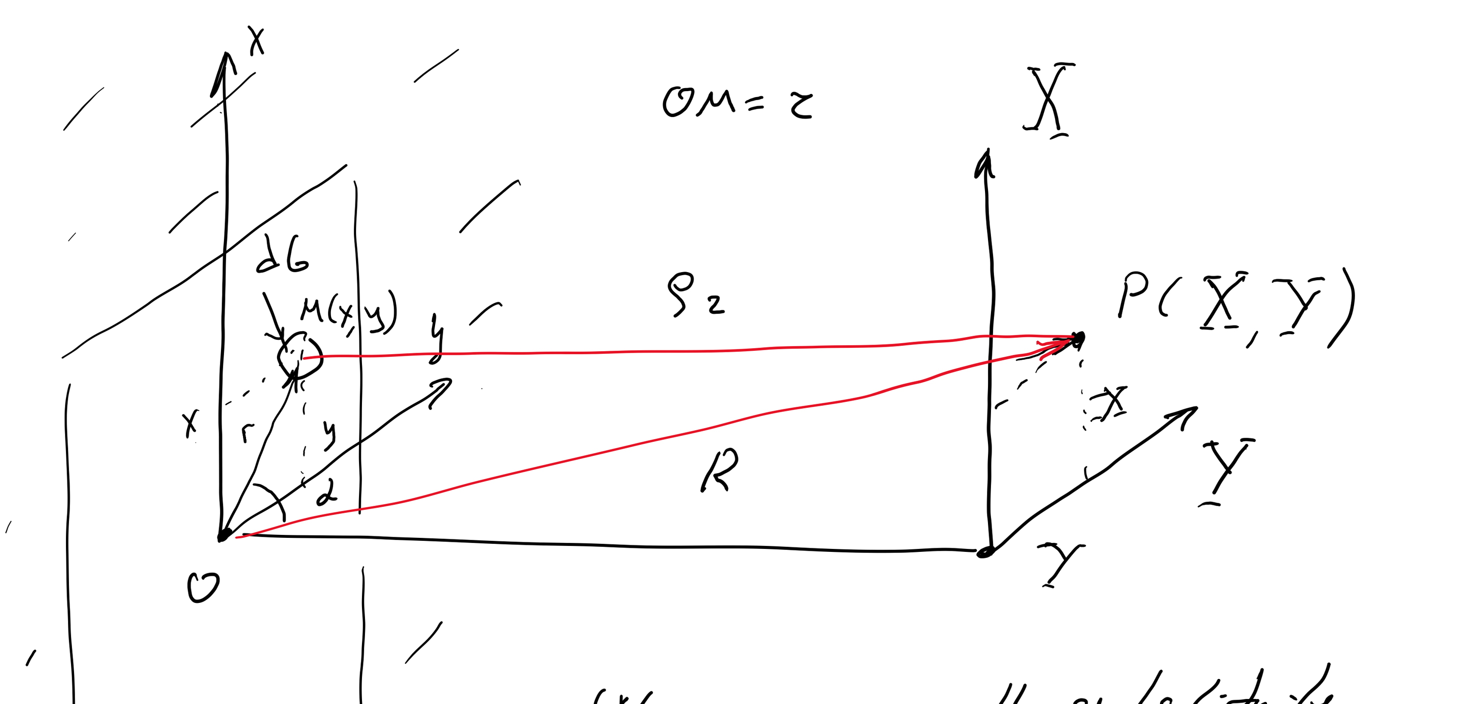
Fraunhofer diffraction on two-dimensional structures.

Fraunhofer diffraction on 2D structures means the field will be altered in both x and y dimensions.



We again will substitute source S with σ and will select single secondary source dσ at point $M(x, y)$. We will look for the field amplitude on screen surface at point $P(X, Y)$.

Similar to the slit problem, we can simplify:

$$E_p = \int_\sigma E(M) \frac{e^{ik\rho_2}}{\rho_2} d\sigma$$

$$E(M) \equiv E(x, y) \qquad d\sigma = dx\,dy$$

Again

$$\rho_2^2 = R^2 + (x^2+y^2) - 2R\sqrt{x^2+y^2}\cos\alpha$$

$$\cos\alpha = \frac{\bar{r}\bar{R}}{rR} = \frac{xX + yY}{R\sqrt{x^2+y^2}}$$

$$\Downarrow$$

$$\rho_2^2 = R^2 + x^2 + y^2 - 2(xX + yY)$$

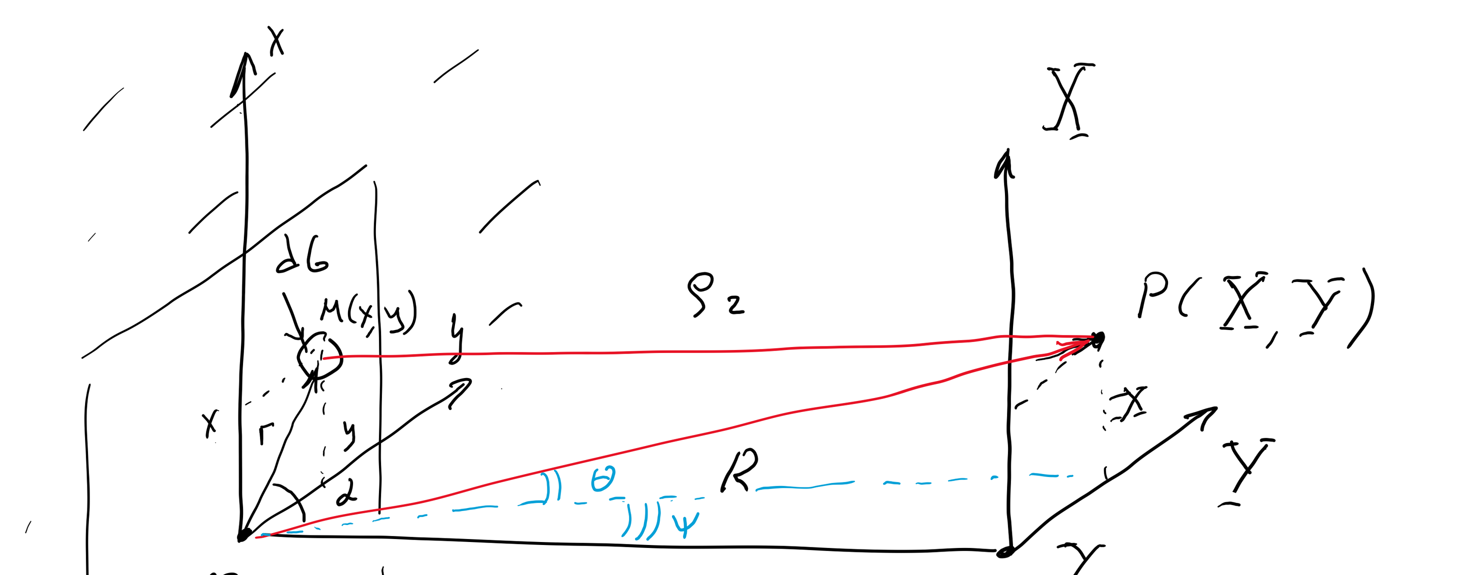
$$\rho_2 = \sqrt{\ } \approx R\left(1 + \underbrace{\frac{x^2}{2R^2}}_{0} + \underbrace{\frac{y^2}{2R^2}}_{0} - x\frac{X}{R^2} - y\frac{Y}{R^2}\right)$$

$$\begin{cases} X \sim 10^{-4}\,m \\ X \sim 10^{-2}\,m \\ R \sim 1\,m \end{cases}$$

This is again indication of transition into the far zone of diffraction which is Fraunhofer diffraction.

Let's introduce the following

$$\frac{X}{R} = \sin\theta \qquad \frac{Y}{R} = \sin\psi$$



$$\rho_2 = R - x\sin\theta - y\sin\psi$$

Hence,

$$\frac{e^{ik\rho_2}}{\rho_2} = \frac{e^{ikR}e^{-ik\sin\theta x - ik\sin\psi y}}{R - \underbrace{x\sin\theta}_{0} - \underbrace{y\sin\psi}_{0}}$$

Though in denominator we can assume small terms zero, in exponent we cannot make such assumption as $k \sim 10^7$.

$$k\sin\theta = k_x \qquad k\sin\psi = k_y$$

$$E_p \sim \iint_{-\infty}^{+\infty} E(x, y) e^{-i(k_x x + k_y y)} dx\,dy = E(k_x, k_y)$$

This is 2D Fourier transformation

$$I_p \sim |E(k_x, k_y)|^2$$

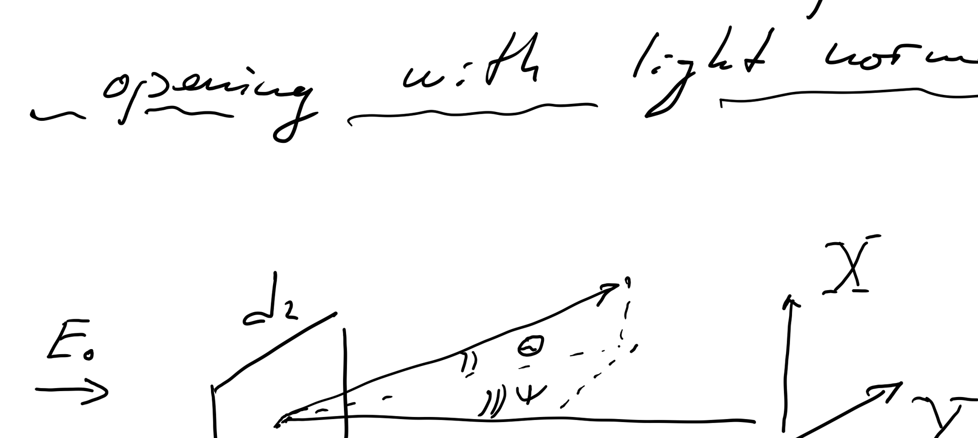
If we know angular spectrum $E(k_x, k_y) \Rightarrow$

$\Rightarrow$ we can find $E(x, y)$

$$E(x, y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} E(k_x, k_y) e^{i(k_x x + k_y y)} dk_x\,dk_y$$

This is very interesting as it shows if we know distribution of field on the screen, we can reconstruct the form of the opening.

Let's see an example of rectangular opening with light normal incidence



Let's use our algorithm.

$$E(x, y) = \begin{cases} E_0, & -\frac{d_1}{2} \le x \le \frac{d_1}{2}; \; -\frac{d_2}{2} \le y \le \frac{d_2}{2} \\ 0, & \text{in remaining points} \end{cases}$$

$$I_p \to E_p \to E(k_x, k_y)$$

$$E(k_x, k_y) = E_0 \int_{-\frac{d_1}{2}}^{\frac{d_1}{2}} \int_{-\frac{d_2}{2}}^{\frac{d_2}{2}} e^{-i(k_x x + k_y y)} dx\,dy =$$

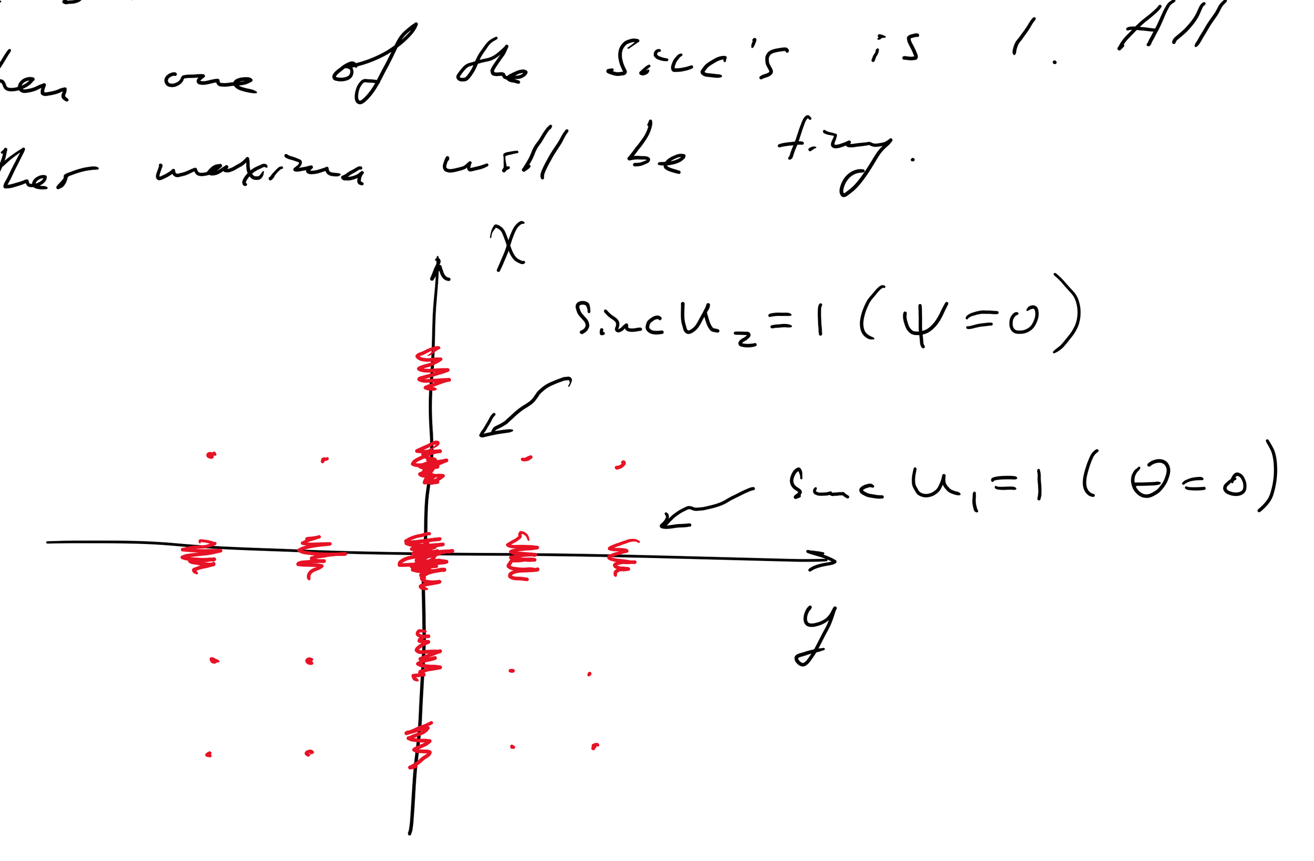
$$= E_0 \underbrace{\int_{-\frac{d_1}{2}}^{\frac{d_1}{2}} e^{-ik_x x} dx}_{\text{sinc}\,u_1} \underbrace{\int_{-\frac{d_2}{2}}^{\frac{d_2}{2}} e^{-ik_y y} dy}_{\text{sinc}\,u_2}$$

$$\begin{cases} u_1 = \frac{kd_1\sin\theta}{2} \\ u_2 = \frac{kd_2\sin\psi}{2} \end{cases}$$

Hence,

$$\boxed{I_p = I_0\,\text{sinc}^2\left(\frac{kd_1\sin\theta}{2}\right) \cdot \text{sinc}^2\left(\frac{kd_2\sin\psi}{2}\right)}$$

I_0 is an intensity in the middle of diffraction pattern, when $\theta = 0$ and $\psi = 0$. We see that maximum intensity will be when one of the sinc's is 1. All other maxima will be tiny.



Demonstration
